Section 9.4 *Comparisons of Series*

In this section we will still be studying series with positive terms. We will need to use our knowledge of the convergence and divergence characteristics of geometric series, harmonic series, *p*-series, and telescoping series to understand the convergence and divergence characteristics of series that do not fall within these categories. In general, we will be looking for series that are algebraically similar to the series we want to study, and we will try to understand convergence based on a comparison of two series, one of which has known convergence and divergence properties.

THEOREM 9.12 Direct Comparison Test

Let $0 < a_n \le b_n$ for all n.

1. If $\sum_{n=1}^{\infty} b_n$ converges, then $\sum_{n=1}^{\infty} a_n$ converges.

2. If $\sum_{n=1}^{\infty} a_n$ diverges, then $\sum_{n=1}^{\infty} b_n$ diverges.

1. If the "larger" series converges, then the "smaller" series converges.
2. If the "smaller series diverges, then the "larger" series diverges.

Ex. 1: Determine the convergence or divergence of the series: $\sum_{n=1}^{\infty} \frac{1}{4\sqrt[3]{n}-1}$

Ex. 2: Determine the convergence or divergence of the series: $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n^3+1}}$

THEOREM 9.13 Limit Comparison Test

Suppose that $a_n > 0$, $b_n > 0$, and

$$\lim_{n\to\infty} \left(\frac{a_n}{b_n}\right) = L$$

where L is *finite and positive*. Then the two series $\Sigma\, a_n$ and $\Sigma\, b_n$ either both converge or both diverge.

Ex. 3: Determine the convergence or divergence of the series: $\displaystyle\sum_{n=1}^{\infty} \frac{5n-3}{n^2-2n+5}$

Ex. 4: Determine the convergence or divergence of the series: $\sum_{n=1}^{\infty} \tan\left(\frac{1}{n}\right)$

Ex. 5: Determine the convergence or divergence of the series: $\sum_{n=1}^{\infty} \frac{2}{3^n - 5}$

Ex. 6: Determine the convergence or divergence of the series: $\displaystyle\sum_{n=1}^{\infty}\left[\frac{1}{n+1}-\frac{1}{n+2}\right]$